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# AN EXAMINATION OF BUBBLE SIZES FOUND IN LOCAL BOILING HEAT TRANSFER EXPERIMENTS

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Andrew D. Whitehead

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# AN EXAMINATION OF BUBBLE SIZES FOUND IN LOCAL BOILING HEAT TRANSFER EXPERIMENTS

by

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Submitted in Partial Fulfillment of the
Requirements for the degree of

MASTER OF SCIENCE

at the

Massachusetts Institute of Technology

(1955)

Signature of Author ____________________

Department of Chemical Engineering, July 22, 1955

Signatures of Professors
in Charge of Research ____________________

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Signature of Chairman of
Department - Committee on
Graduate Students ____________________

# An Examination of Bubble Sizes Found in Local Boiling Heat Transfer Experiments

by

Andrew D. Whitehead

Submitted to the Department of Chemical Engineering on 22 July, 1955 in partial fulfillment of the requirements for the degree of Master of Science in Nuclear Engineering.

## ABSTRACT

Photographic examination of the bubbles occuring in Local Boiling Heat Transfer experiments has led to the conclusion that for distilled, degassed water and a stainless steel surface, the bubbles prefer one particular size, -2.86 mils diameter. A model or mechanism for vapor formation is introduced and it is shown that this model supports the observed phenomena of a preferred bubble size. Use is made of the result of the analysis as a condition for boiling, and a possible explanation of the burnout process.

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28391

# ACKNOWLEDGEMENTS

This thesis was carried out under the supervision of Professor W. M. Rohsenow and Professor J. A. Clark, to whom I am indebted for their advice, intelligent criticism, and encouragement. I wish to thank Peter Griffith for the many hours he spent with me in discussion of the very complex problems of bubble formation.

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## 1. Introduction

For some years now there have been considerable efforts made to take advantage of a heat transfer process known as Local Boiling Heat Transfer. It is characterized by a large difference in temperature between the heated surface and the bulk of the fluid. In general, the temperature of the fluid is well below the saturation temperature for the fluid pressure while the heated surface is a few degrees above the saturation temperature. The boiling is confined to the vicinity of the surface. The bubbles that do form condense in the cooler bulk fluid and there is not net vapor generation. The process can immensely increase the heat transfer rate without any appreciable increase in temperature difference between heated surface and bulk fluid.

In the course of some experimental investigations to determine density variations due to local boiling, some several thousand pictures were made of the vapor bubbles formed on the heated surfaces, under a wide range of conditions. At considerable effort, the bubbles on many of these pictures were counted and classified as to size.

The object of this paper is to examine these 'bubble counts' to obtain some further understanding of 'Local Boiling Heat Transfer'.

## 2. Presentation of 'bubble count' data.

### a. Source of data.

As described in reference (H-1), George Henry, in 1953, was able to photograph boiling on an electrically heated surface over which distilled, degassed water was flowing at various combinations of pressures - (500, 1000, 1500 psia) -, velocities - (20, 30 ft. per second) -, subcoolings - ($0^\circ$- $150^\circ$ F) -, and rates of heat transfer - (approximately 10 - 50% required for burnout). The flow channel was a square section .500 inches on a side. The water flowed vertically upward through this channel. The heated surface was of stainless steel and 3.00 inches long and 3/8 inch wide. These conditions resulted in fully developed turbulent flow in the region of interest. The boiling that occurred was observed to be confined to a layer next to the heated strip. Bubbles were observed to form on the surface and to collapse without apparently moving from the surface. This type of boiling is called 'local boiling' and is characteristic of boiling under conditions of highly turbulent flow of subcooled water over heated surfaces.

A 4 by 5 inch laboratory still camera was focused on the heater strip in conjunction with a single flash light source whose period of illumination was about 2 microseconds.

A complete description of the apparatus used and the experimental techniques employed is given in references (H-1). References (M-1) and (M-2) made further use of the apparatus and refined the techniques used in (H-1). Since in this paper little exact use is made of any data, it is believed unnecessary to re-

count the experimental procedures in detail.

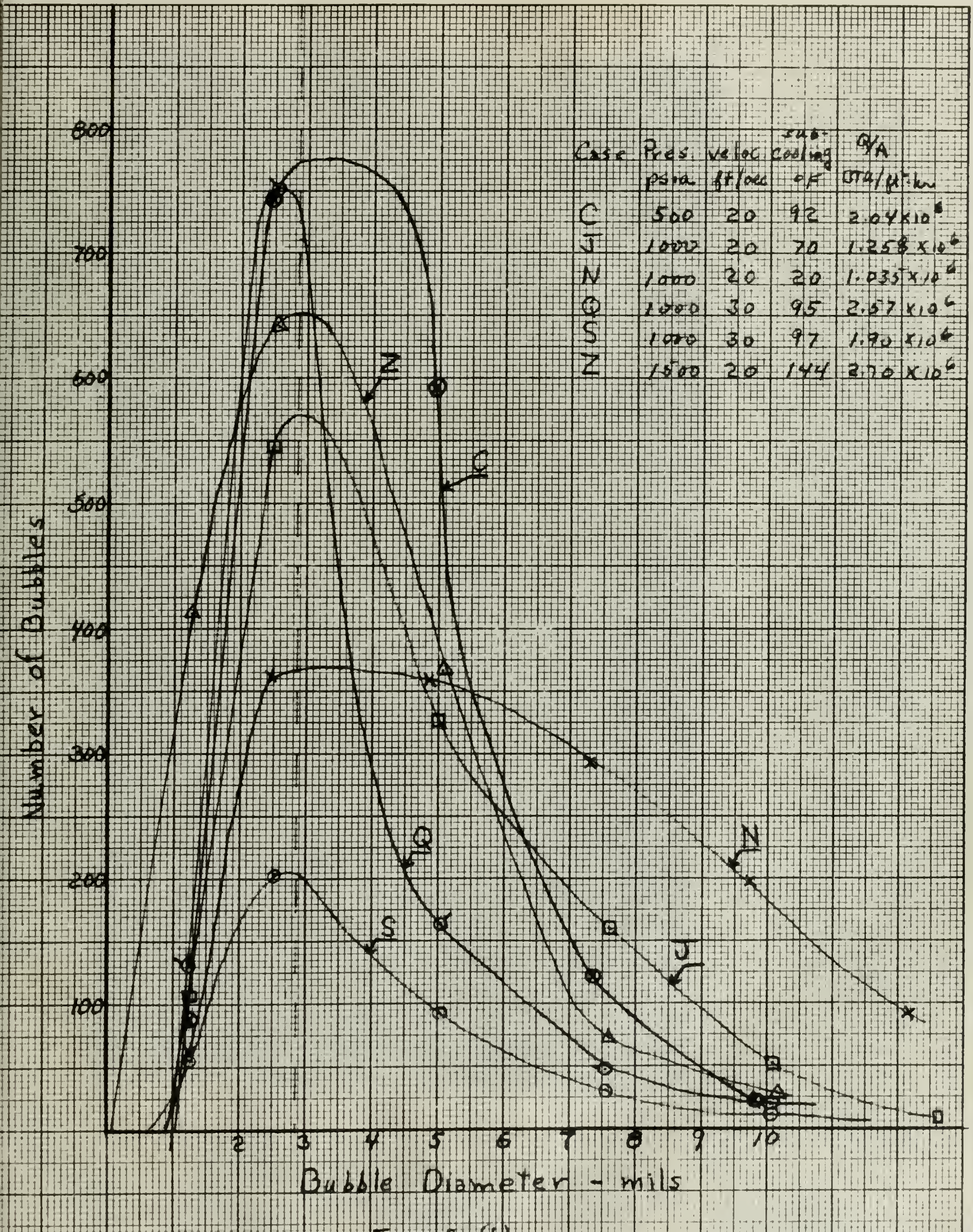
For a particular set of variables, system pressure, bulk temperature, velocity, and heat flux density, ten pictures were taken. Four of these ten were selected as representative and analyzed. To reduce the time for analyzing the pictures, only a small portion of the heater strip neat its upstream end was used. The size of this section was 1/2 inch in length by 1/2 inch in width. Gauge marks in the channel were used to determine the picture magnification for each picture. This area was subdivided and the number of bubbles counted in each subdivision. Then each bubble of picture size of 0.01 inch diameter and greater was measured to the nearest 0.01 inch. All remaining bubbles were grouped at a picture size of 0.005 inches.

The actual dimensions of the bubble were determined from the magnification factor (approximately 5), and a factor to account for the relation between apparent bubble size and true bubble size (experimentally determined to be .834). This is worked out in reference (H-1) and corrected in reference (M-1). With measurements made to the nearest 0.01 inch, a magnification factor of 5 and a conversion factor of .834, it is apparent that the resolution in determining the actual size of the bubble is of the order of 2.5 mils.

The results are tabulated in appendix I as numbers of bubbles at each bubble size for the particular conditions in force.

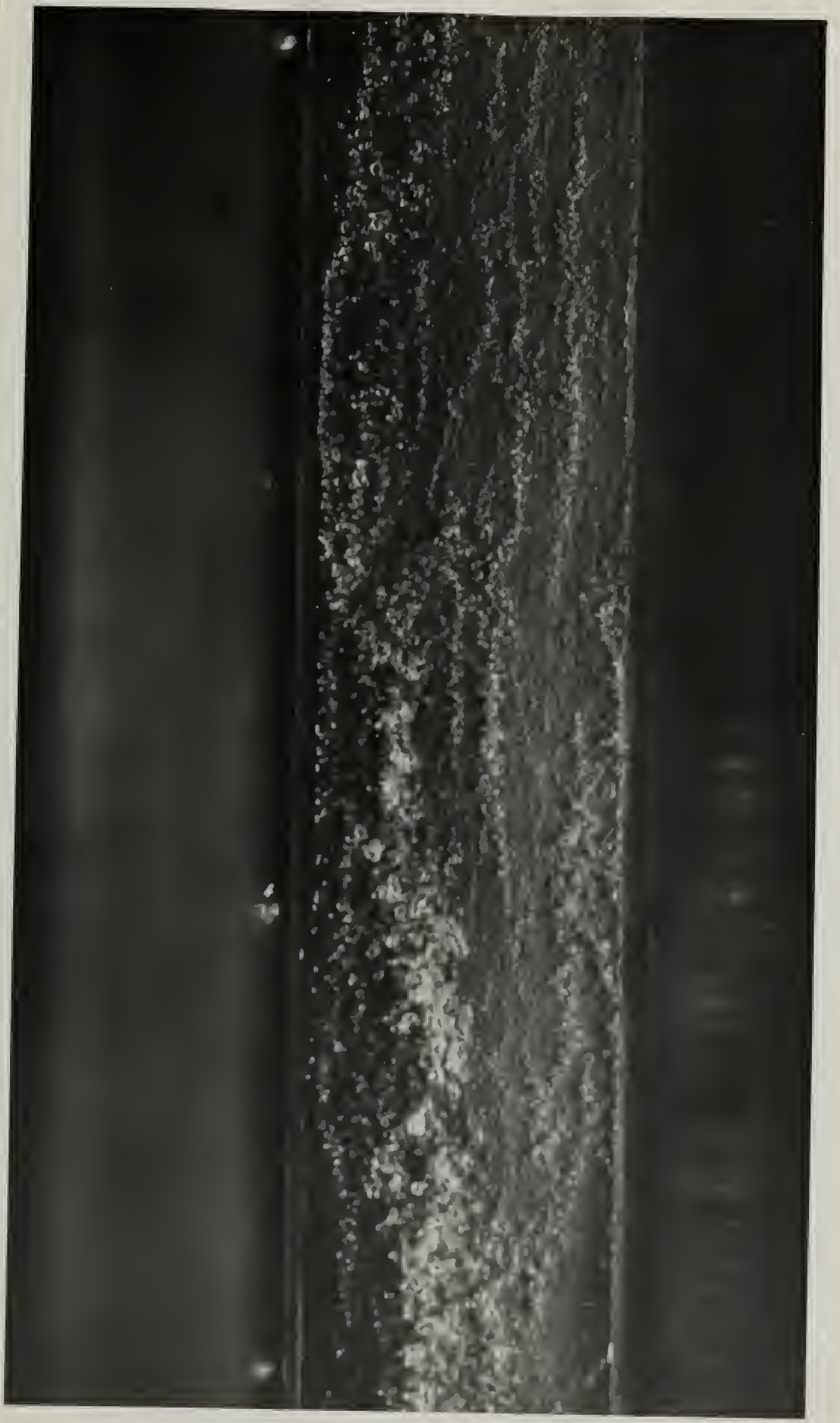
b. Discussion of Data.

Several representative sets of data from appendix I have been plotted in figure (1). One of the four pictures that was used to obtain each set of data is shown in figures (2) through (7).



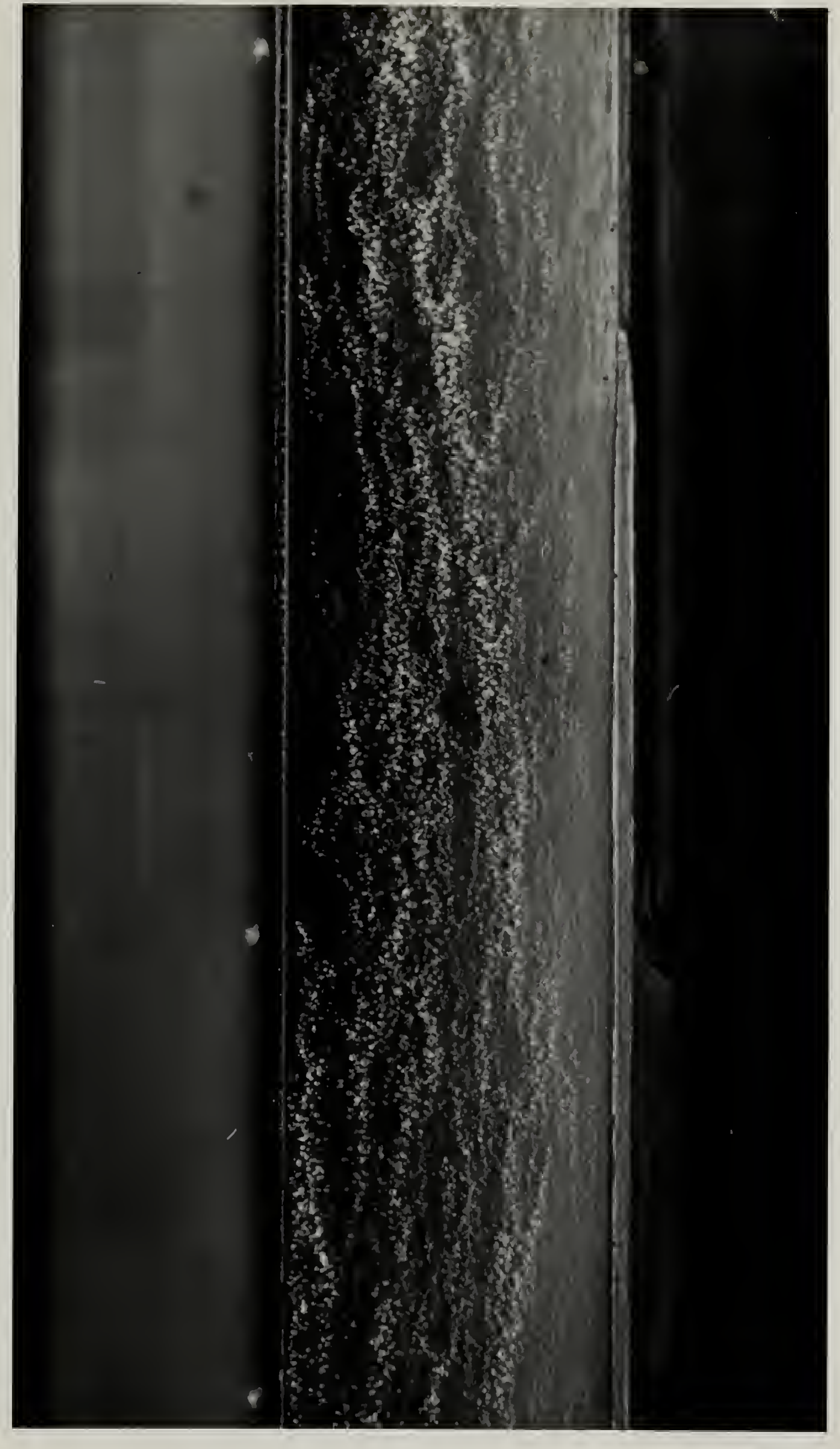
| Case | Pres. psia | veloc ft/sec | sub-cooling °F | Q/A BTU/ft²-hr |
|---|---|---|---|---|
| C | 500 | 20 | 92 | $2.04 \times 10^6$ |
| J | 1000 | 20 | 70 | $1.258 \times 10^6$ |
| N | 1000 | 20 | 20 | $1.035 \times 10^6$ |
| Q | 1000 | 30 | 95 | $2.57 \times 10^6$ |
| S | 1000 | 30 | 97 | $1.90 \times 10^6$ |
| Z | 1500 | 20 | 144 | $2.70 \times 10^6$ |

Figure (1)



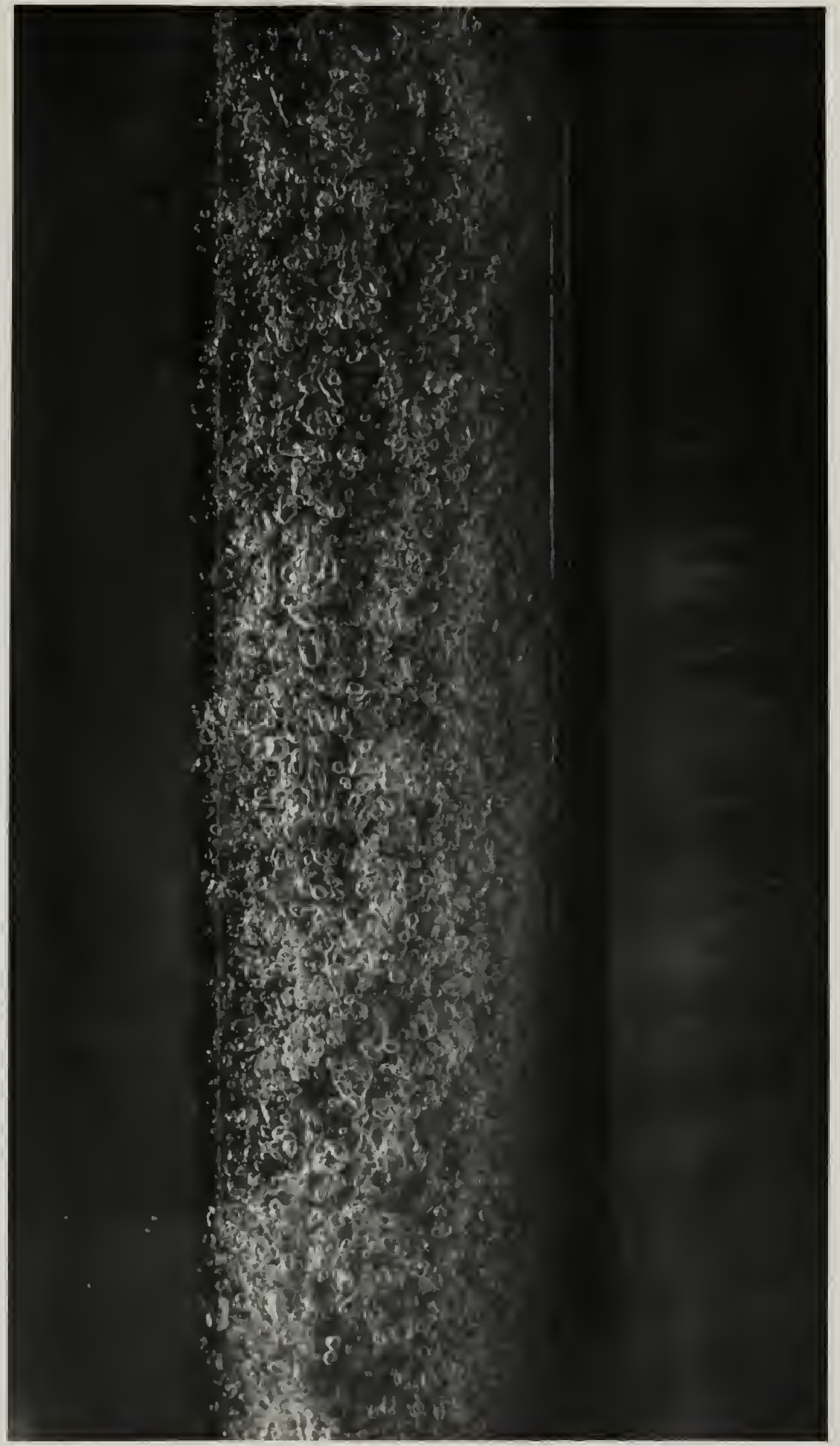
| Case | Pressure | Velocity | Subcooling | q/A |
|---|---|---|---|---|
| C | 500 psia | 20 ft/sec. | 92 °F | $2.04 \times 10^6$ BTU/hr-ft$^2$ |

Figure (2)



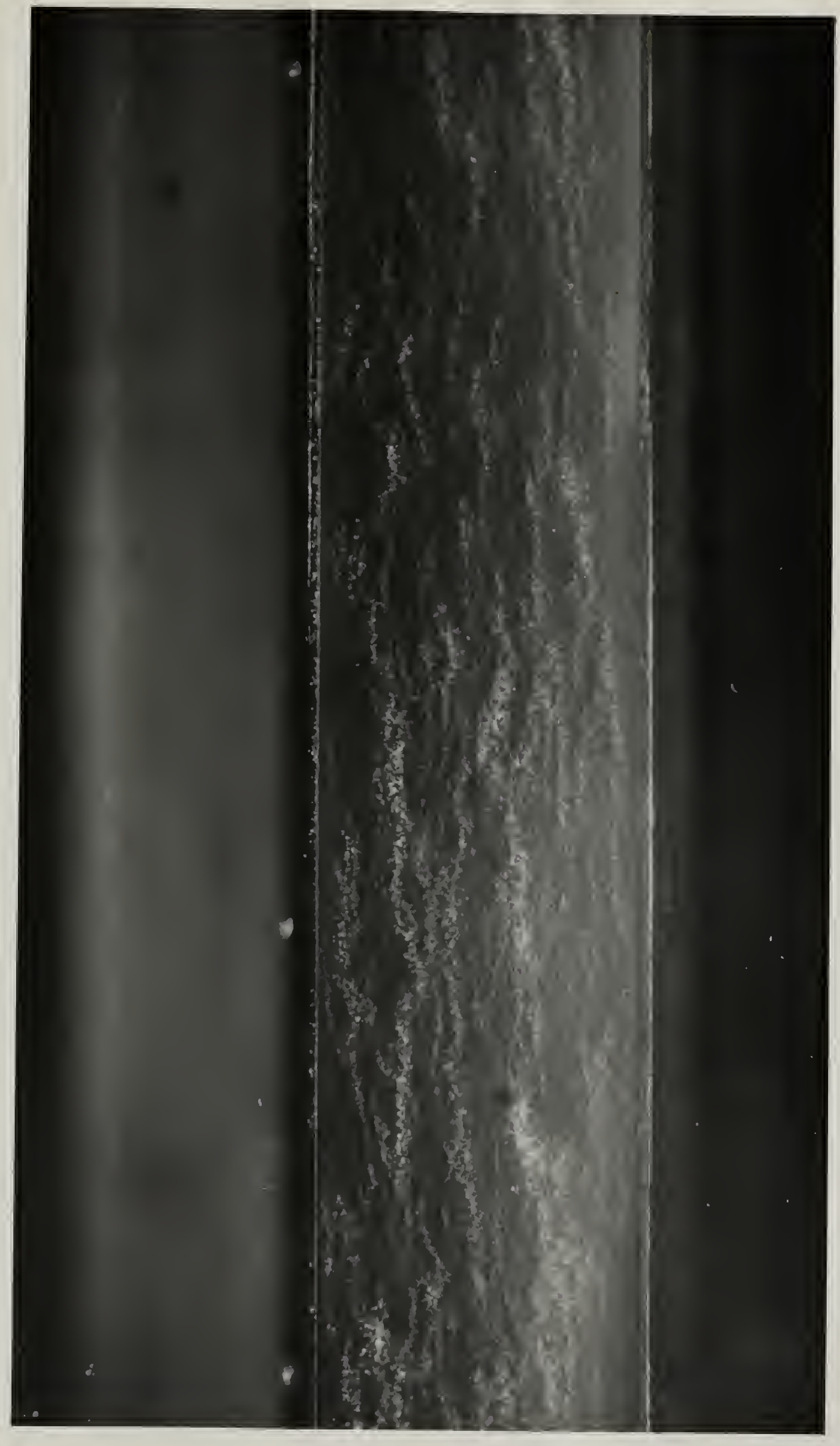
| Case | Pressure | Velocity | Subcooling | Q/A |
|---|---|---|---|---|
| J | 1000 psia | 20 ft/sec. | 70°F | $1.258 \times 10^6$ BTU/hr-ft$^2$ |

Figure (3)



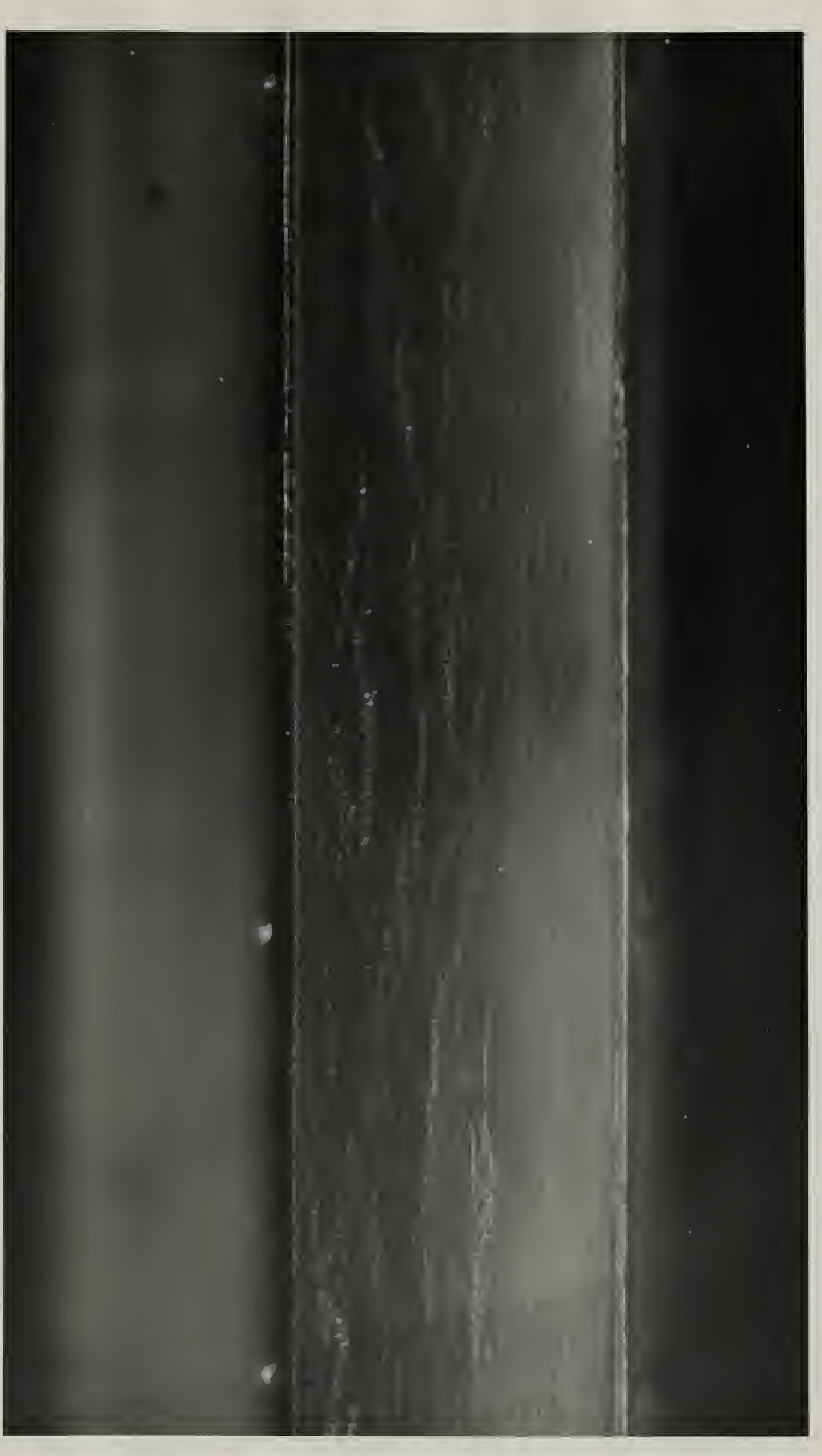
| Case | Pressure | Velocity | Subcooling | Q/A |
|---|---|---|---|---|
| N | 1000 psia | 20 ft/sec. | 20°F | $1.035 \times 10^6$ BTU/hr-ft$^2$ |

Figure (4)



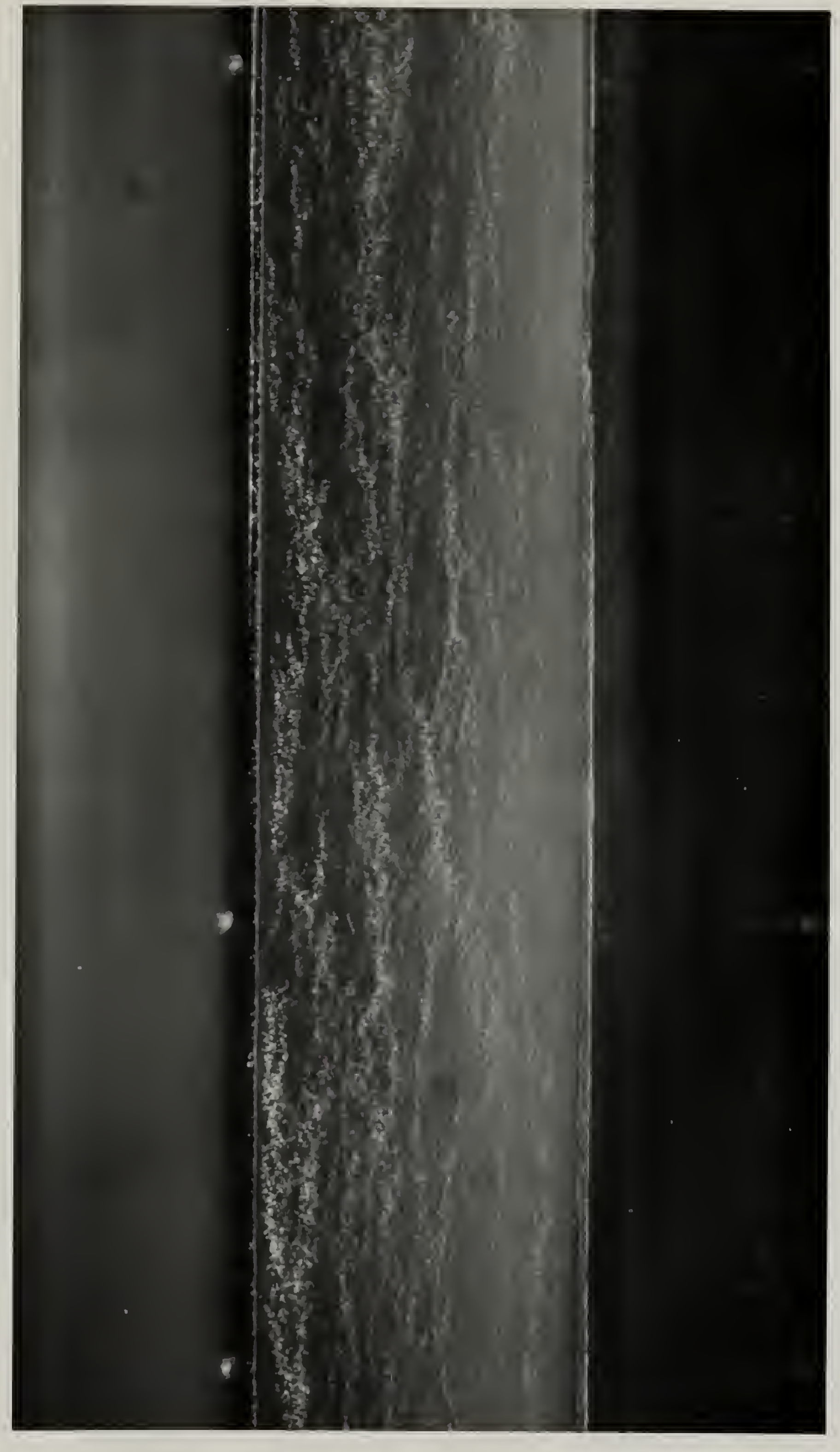
| Case | Pressure | Velocity | Subcooling | q/A |
|---|---|---|---|---|
| 2 | 1000 psia | 30 ft/sec. | 95 °F | $2.57 \times 10^6$ BTU/hr-ft$^2$ |

Figure (5)



| Case | Pressure | Velocity | Subcooling | q/A |
|---|---|---|---|---|
| 8 | 1000 psia | 30 ft/sec. | 97 °F | $1.90 \times 10^6$ BTU/hr-ft$^2$ |

Figure (6)



| Case | Pressure | Velocity | Subcooling | Q/A |
|---|---|---|---|---|
| Z | 1500 psia | 20 ft/sec. | 144 °F | $2.70 \times 10^6$ BTU/hr-ft$^2$ |

Figure (7)

The pictures appear to show considerable variations in size and numbers of bubbles with changes in pressure, velocity, subcooling, and rate of heat transfer. At first glance, the curves of figure (1) show similar degrees of variation. However, the really striking thing about the curves is that they all tend to peak at about 2.86 mils diameter. All of the data in appendix I has been plotted and the diameter at which the peak occurs noted. This is presented in Table I, along with the deviations from a mean value of 2.86 mils. Bearing in mind that the resolution used in measuring bubble diameter was about 2.5 mils, it appears that, for the experimental cases investigated, the majority of bubbles at any instant are in the approximate range of 2.1 to 3.6 mils diameter.

In reference (G-1), Gunther made a photographic study of boiling under forced convection conditions at various flow rates, degrees of subcooling, heat fluxes, but at essentially atmospheric pressure. He concluded that bubble size increased with decreasing velocity, with decreasing subcooling, and moderately increased with decreasing heat transfer rate. His conclusions were made on the measurement of relatively few bubbles. His conclusions do not appear to be borne out by the more exhaustive measurements presented in this paper.

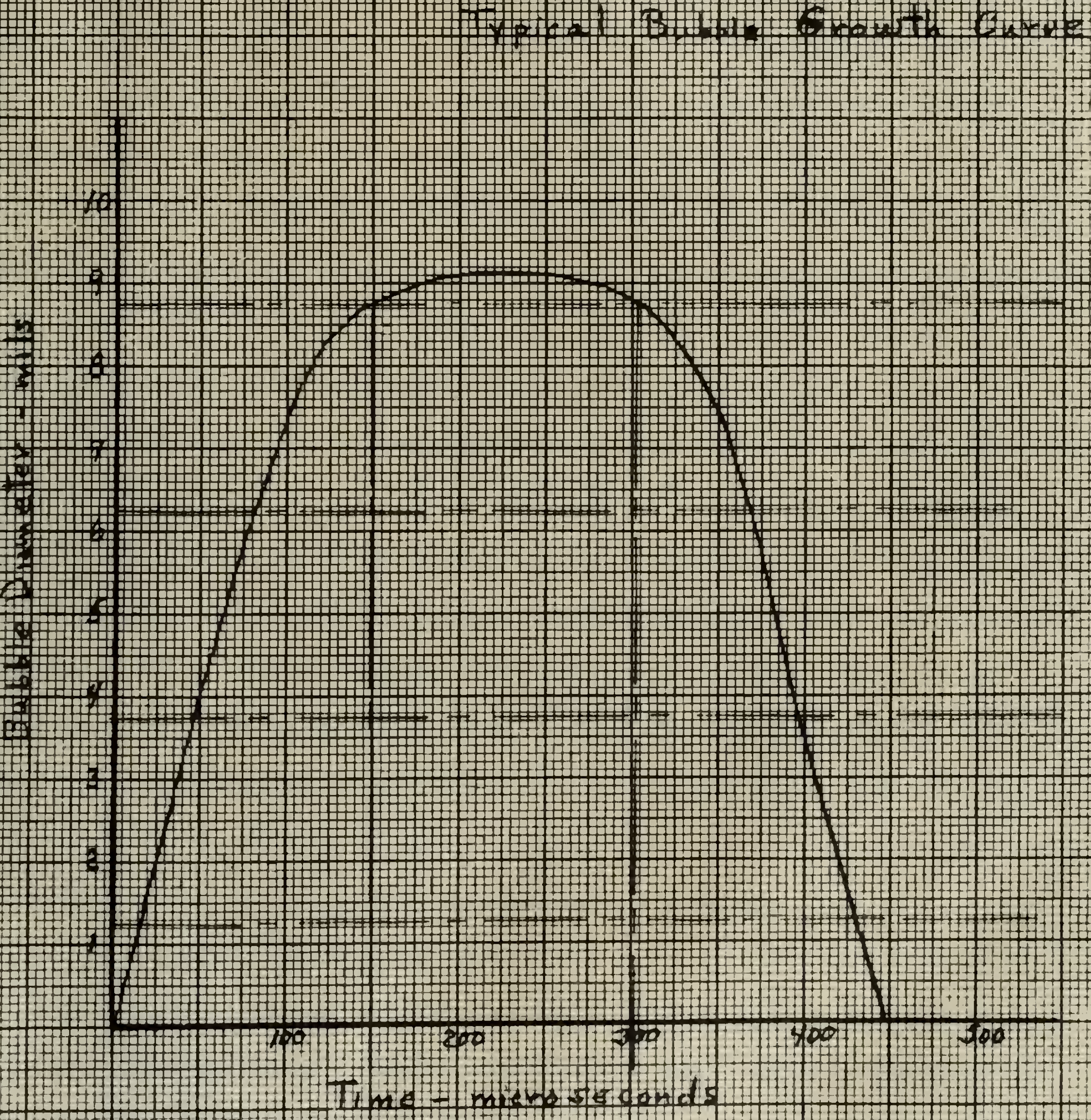
Gunther, in the same paper (G-1), provided several graphs of bubble size as a function of time. A curve of the general shape of Gunther's is given in figure (8). Blocking it off in equal increments of size (which is the way the bubbles were measured in the data presented) makes it quite obvious that a

## TABLE 1

| Case | Pressure psia | Velocity ft/sec | Q/A BTU/$ft^2$-hr | Subcooling °F | Diameter mils | Deviation mils |
|---|---|---|---|---|---|---|
| A | 500 | 20 | $2.34 \times 10^6$ | 117 | 2.8 | -.06 |
| B | 500 | 20 | $2.02 \times 10^6$ | 117 | 2.8 | -.06 |
| C | 500 | 20 | $2.04 \times 10^6$ | 92 | 3.2 | +.34 |
| D | 500 | 20 | $1.705 \times 10^6$ | 92 | 3.0 | +.14 |
| E | 500 | 20 | $1.43 \times 10^6$ | 92 | 2.9 | +.04 |
| F | 1000 | 20 | $2.155 \times 10^6$ | 95 | 3.4 | +.54 |
| G | 1000 | 20 | $1.821 \times 10^6$ | 95 | 3.0 | +.14 |
| H | 1000 | 20 | $1.516 \times 10^6$ | 95 | 2.7 | -.16 |
| I | 1000 | 20 | $1.54 \times 10^6$ | 70 | 3.5 | +.64 |
| J | 1000 | 20 | $1.258 \times 10^6$ | 70 | 2.8 | -.06 |
| K | 1000 | 20 | $1.285 \times 10^6$ | 45 | 3.0 | +.14 |
| L | 1000 | 20 | $1.022 \times 10^6$ | 45 | 4.0 | +1.14 |
| M | 1000 | 20 | $.799 \times 10^6$ | 45 | 3.2 | +.34 |
| N | 1000 | 20 | $1.035 \times 10^6$ | 20 | 3.2 | +.34 |
| O | 1000 | 20 | $.810 \times 10^6$ | 20 | 3.0 | +.14 |
| P | 1000 | 20 | $.610 \times 10^6$ | 20 | 3.0 | +.14 |
| Q | 1000 | 30 | $2.57 \times 10^6$ | 95 | 2.5 | -.36 |
| R | 1000 | 30 | $2.22 \times 10^6$ | 97 | 2.0 | -.86 |
| S | 1000 | 30 | $1.90 \times 10^6$ | 97 | 3.0 | +.14 |
| T | 1000 | 30 | $1.832 \times 10^6$ | 70 | 2.4 | -.46 |
| U | 1000 | 30 | $1.545 \times 10^6$ | 70 | 2.6 | -.26 |
| V | 1000 | 30 | $.846 \times 10^6$ | 20 | 2.6 | -.26 |
| W | 1000 | 30 | $.614 \times 10^6$ | 20 | 2.6 | -.26 |
| X | 1000 | 30 | $.452 \times 10^6$ | 10 | 2.6 | -.26 |
| Y | 1000 | 30 | $.321 \times 10^6$ | 7 | 2.7 | -.16 |

TABLE I (continued)

| Case | Pressure psia | Velocity ft/sec | Q/A BTU/ft$^2$-hr | Subcooling $^\circ$F | Diameter mils | Deviation mils |
|---|---|---|---|---|---|---|
| Z | 1500 | 20 | $2.70 \times 10^6$ | 144 | 2.7 | -.16 |
| AA | 1500 | 20 | $2.27 \times 10^6$ | 146 | 2.5 | -.36 |
| BB | 1500 | 20 | $1.97 \times 10^6$ | 94 | 2.6 | -.26 |
| CC | 1500 | 20 | $1.669 \times 10^6$ | 69 | 2.7 | -.16 |



Typical Bubble Growth Curve

Figure (8)

bubble spends the majority of its life time at or nearly at its maximum size. Therefore, a picture which depicts a one or two microsecond instant will catch most of the bubbles at their maximum size. The data presented is really proportional to the maximum sizes attained by the bubbles. The peaks at a bubble size of 2.86 mils represent fully developed bubbles and are not the result of a picture catching a great number of bubbles in a growth or decay phase.

There are, for many cases, large numbers of relatively big bubbles counted. A close lock at figure (4) suggests the cause. What were counted as large bubbles were most likely mergers of several smaller bubbles. The very fact that even in such a case at that in figure (4) - case K - the bubble count shows a peak in the vicinity of 2.86 mils tends to support this theory.

In summary, the data suggests that for highly turbulent flow of cold water over a heated surface, bubbles prefer to grow to one particular size. That this particular size is a constant for a wide range of conditions and rates of heat transfer is also strongly suggested.

## 3. Proposed Model for the Formation of a Bubble.

### a. Temperature distribution.

In the experiments under consideration, highly turbulent water is flowing over a heated surface. Much work has been done in studying the velocity distribution for isothermal turbulent flow and is concisely summarized by McAdams, reference (M-3). Generally speaking, three regions of flow are usually described; a laminar sublayer next to the surface in which the flow is

streamline, a buffer layer in which the motion of the water oscillates between streamline and turbulent flow, and the bulk of the fluid in which the flow is completely turbulent. Experimental work, particularly by Nakuradse, has led to equations for the velocity distribution. From these, equations for the eddy diffusivity of momentum can be written. Now if we consider that the boundary is heated, we can say that heat is conducted through the laminar boundary layer by molecular conduction, through the buffer layer by both molecular conduction and mechanical mixing, and through the bulk by mechanical mixing. Equations similar to those for eddy diffusivity of momentum can be written for the eddy diffusivity of heat. The success of Martinelli and others in utilizing this analogy leads to the prediction that the temperature distribution through the water is very nearly the same as the velocity distribution. For the purpose of this paper we will consider two regions of flow and their temperature distribution. We will assume that in the laminar sublayer the temperature has a linear distribution varying from the boundary temperature to the bulk temperature, and that the bulk temperature is constant.

b. Adiabatic formation of a bubble.

We will assume that the bubble will originate at a point on a heated surface. Available to this point is the energy from the water around it. The model we propose is that a hemisphere of water on the heated surface with a temperature distribution as given in section 2. a. change adiabatically from this state to a state composed of part vapor and part liquid, all now at saturation temperature corresponding to the pressure

of the system. There are several reasons for choosing this model. First, bubbles are known to originate on surfaces. One of the reasons for this is that the presence of minute curvatures on the surface lessen the energy requirements for bubble formation. This is in accordance with Gibbs requirement for thermo dynamic equilibrium as regards pressures of a two phase system:

$p'' - p' = (c_1 + c_2)\sigma$ Secondly, as pointed out in section 3.1, there is almost certainly a slow moving layer of water next to the boundary through which a temperature gradient probably exists. Thirdly, a hemisphere is a reasonable shape as there appears no reason to suppose a preferential direction for the flow of energy to the selected point. In the fourth place, since the formation of a bubble is known to be a rapid process, it can well be assumed to be an adiabatic one. And lastly, it seems correct to state that at its maximum size, a bubble will have attained thermal equilibrium with its surrounding.

## 4. Application of Gibbs Stability Criterion to Model.

In the proposed model, water existing in one state suddenly ceases to exist in this state and appears in an entirely different state. According to Gibbs, reference (G-2): "For the equilibrium of any isolated system it is necessary and sufficient that in all possible variations of the state of the system which do not alter its energy, the variation of its entropy shall either vanish or be negative." Conversely, if our model is a possible one, the entropy of the water in its second state must be greater than its entropy in the first state.

Gibbs criterion certainly implies that the most probable state of an isolated system is the one for which its entropy is greatest. There seems no reason why this cannot be interpreted to say that any non-equilibrium state of a system can spontaneously shift to a state of higher entropy. There seems no reason to require that the second or new state be an equilibrium state.

We have taken for our system the mass of a hemisphere of water on the heated surface. Such a system, as it occurs in the experimental setup, is one of an infinite number of such systems. It is not an isolated system. However, if we assume that during the transition from one state to another, the system suffers no gain or loss of energy, then we can still apply Gibbs criterion.

We can write this assumption as an equation

1.) $$dE - dQ + dW = 0$$

Since we have postulated an adiabatic change of state:

2.) $$dE + dW = 0$$

or

3.) $$dA = 0$$

For the transition to take place,

4.) $$M_g s_g + M_f s_f - \int_0^M c_p \ln T \, dm \geq 0$$

or the entropy of the vapor plus the entropy of the liquid in the second state must be greater than the entropy of the liquid in the first state. For simplicity, we have neglected the entropies of the interfaces.

From our model we can write the temperature gradient:

5.) $$T = T_w - \left(\frac{T_w - T_s}{x_s}\right) z$$

Since our system is a mass of water we can write:

6.) $$M_g + M_f = \int_0^M dm = M = \frac{2}{3}\pi \rho_f R^3$$

The enthalpies of the two states being equal gives equation

7.) $$M_g h_g + M_f h_f = \int_0^M h\, dm = \int_0^R \rho_f c_p \left(T_w - \left(\frac{T_w - T_s}{x_s}\right) z\right)(R^2 - z^2)\, dz$$

8.) $$M_g h_g + M_f h_f = \pi \rho_f c_p \left(\frac{2}{3} T_w R^3 - \frac{1}{4}\left(\frac{T_w - T_s}{x_s}\right) R^4\right)$$

Solving equations (6) and (8) for Mg gives:

9.) $$M_g = \frac{\frac{2}{3}\pi h_f \rho_f R^3 - \pi \rho_f c_p \left(\frac{2}{3} T_w R^3 - \frac{1}{4}\left(\frac{T_w - T_s}{x_s}\right) R^4\right)}{h_f - h_g}$$

We can rewrite equation (1) as

10.) $$\Delta S = (M - M_f) s_g + M_f s_f - \int_0^M s\, dm$$

11.) $$\Delta S = - M_f s_{fg} + \int_0^M (s_g - s)\, dm$$

12.) $$\Delta S = M_g s_{fg} - \int_0^M (s - s_f)\, dm$$

13.) $$\Delta S = M_g s_{fg} - \int_0^M c_p \ln T/T_s\, dm$$

14.) $$\int_0^M c_p \ln T/T_s\, dm = \int_0^R \rho_f c_p \pi \left[\ln\left(\frac{T_w}{T_s} - \left(\frac{T_w - T_s}{T_s x_s}\right) z\right)\right](R^2 - z^2)\, dz$$

Rearranging and using the series expansion for ln(x) we can write the integral as:

15.) $$\int_0^R \rho_f c_p \pi (R^2 - z^2)\left[\left(\frac{T_w - T_s}{T_s}\right)\left(1 - \frac{z}{x_s}\right) - \frac{1}{2}\left(\frac{T_w - T_s}{T_s}\right)^2\left(1 - \frac{z}{x_s}\right)^2 + \cdots\right] dz$$

If this integral is worked out it is found that the first term in the brackets produces the true value of the integral almost exactly for all values of R less then $3x_s$ which turns out to include all systems having positive $\Delta S$. Therefore we will write the integral

16.) $$\int_0^R \rho_f c_p \pi (R^2 - z^2)\left[\left(\frac{T_w - T_s}{T_s}\right)\left(1 - \frac{z}{x_s}\right)\right] dz$$

This is readily integrated to:

17.) $$\rho_f c_p \pi \left(\frac{T_w - T_s}{T_s}\right)\left(\frac{2}{3}R^3 - \frac{R^4}{4x_s}\right)$$

Combining this result with equation (6) we can write:

18.) $$\Delta S = \left(1 - \frac{1}{T_s}\right)(T_w - T_s)\left[\frac{s_{fg}\pi \rho_f c_p}{h_{fg}}\right]\left[R^3\right]\left[\frac{2}{3} - \frac{1}{4}\frac{R}{x_s}\right]$$

The first term is almost exactly one and we can write:

19.) $$\Delta S = (T_w - T_s)\left[\frac{s_{fg}\pi \rho_f c_p}{h_{fg}}\right]\left[R^3\right]\left[\frac{2}{3} - \frac{1}{4}\frac{R}{x_s}\right]$$

20.) $$\Delta S = \left(\frac{T_w - T_s}{T_s}\right)(\pi \rho_f c_p)(R^3)\left(\frac{2}{3} - \frac{1}{4} R/x_s\right)$$

Throughout this derivation we have assumed that the density, $\rho_f$, and the heat capacity, $c_p$, of the liquid in its initial state were constants. This is reasonable if the wall temperature, $T_w$, is not very much greater than the saturation temperature, $T_s$.

If we consider a situation in which the wall temperature is constant, above saturation temperature, but below that which causes vigorous boiling, the term $x_s$ which appears in equation

(20) must be a constant. For a given system, a given mass of water, a given hemisphere of water, a given R in equation (20) we will get one value of $\Delta S$. If this value is positive we can say that the given system is in an unstable state. If we plot this function against system size, R, we get the curve of figure (9). This has been normalized.

## 5. Interpretation of Result of Applying Gibbs Criterion.

Bearing in mind the model chosen, we can say that whenever there is a temperature gradient through the water, there exists a *most* unstable system. This is the peak point in figure (9). The model and equation (20) cannot be interpreted to give the conditions necessary for a change of state. They should be interpreted as follows. If the gradient through the water is changing, one particular system has a much greater *probability* of changing state *before* any other system does. That is, one particular system (the one for which $R = 2x_s$) will reach critical unstability first. Critical unstability is probably determined by such factors as roughness of surface, purity of water, wettability of surface, work to produce interfaces, both solid-vapor and liquid-vapor, and many others.

Equation (9) can be differentiated with respect to R and, it is found that $R = 2x_s$ is the necessary condition, for the differentiated equation to equal zero. Thus the system that first reaches critical unstability is also the system that produces the greatest mass of vapor when it changes state. If we substitute $R = 2x_s$ in equation (9) and let

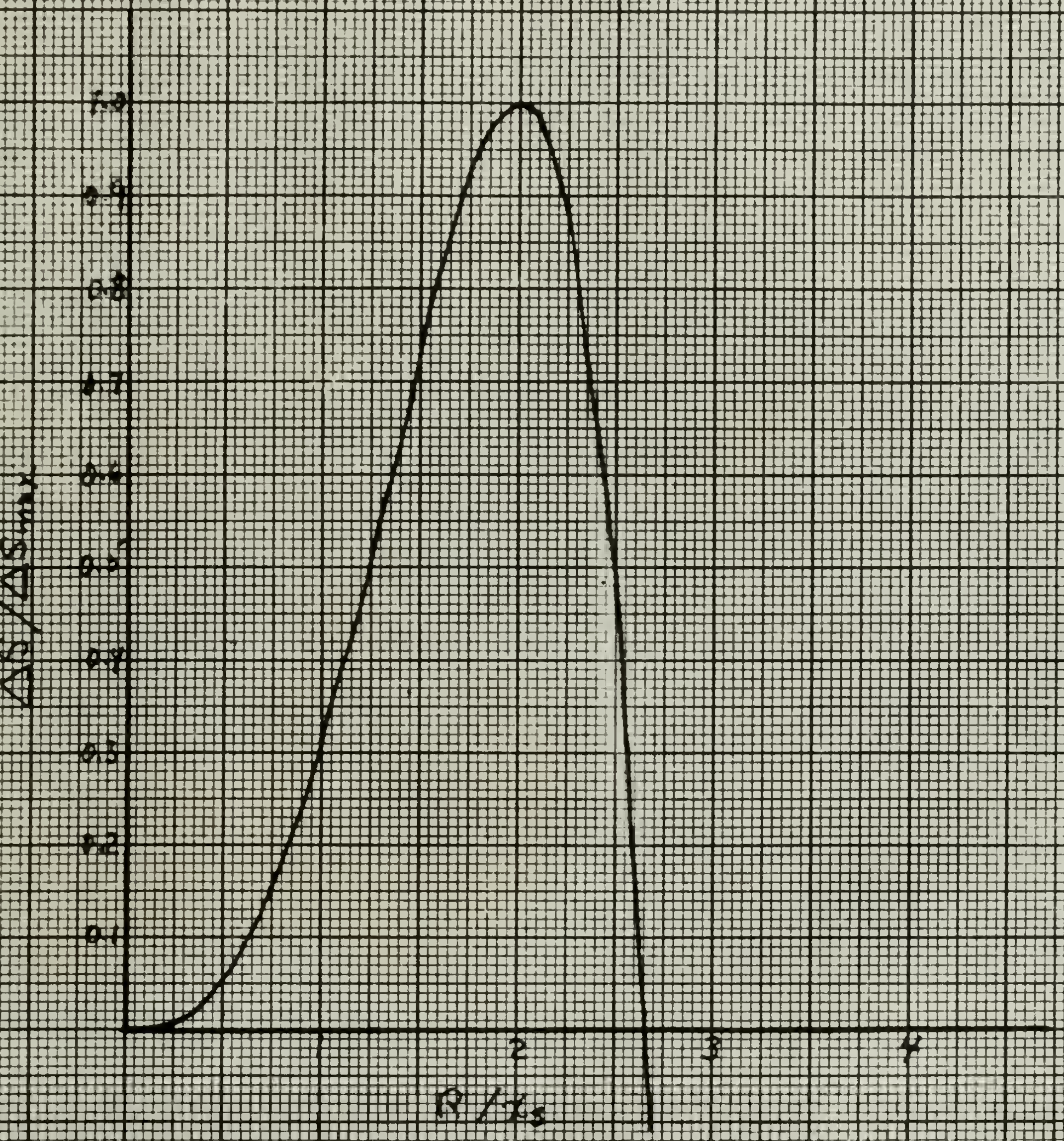


Figure (9)

21.) $$M_g = \frac{4}{3}\pi \rho_g r^3$$

we arrive readily at the result

22.) $$x_s^3 = \frac{h_{fg}\rho_g}{c_p\,\rho_f}\;\frac{r^3}{(T_w - T_s)}$$

From the discussion in section 2, it appears that we could substitute for r the value 1.43 mils, and the equation becomes

23.) $$x_s^3 = \frac{h_{fg}\rho_g}{c_p\,\rho_f}\;\frac{3.92}{(T_w - T_s)}$$

The interpretation to be placed on this equation is as follows. If we can satisfy the equation, boiling will occur. The quantity $x_s$ is dependent on the thickness of the laminar sublayer, which is determined by fluid flow mechanics if there is no boiling. Thus some solutions of equation (23) will just not be possible. That is, if $(T_w - T_s)$ is a very small number, the $x_s$ required by the equation may be so great as to place it outside the laminar sublayer.

In the experiments being considered it has been found, reference (H-1), that $(T_w - T_s)$ is approximately $15^\circ$ F at a pressure of 1000 psia. The value of $h_{fg}\rho_g / c_p\rho_f$ at this pressure is 2.9 in the appropriate units. Therefore, the value of $x_s$ required is approximately 0.91 mils. This must have been a realizable value. It seems a reasonable one.

The action of boiling is generally considered to increase the turbulence near the heated surface. This added turbulence or mixing is felt to account for the greatly increased heat transfer. As the vapor moves away from the surface, liquid from the

surrounding replaces it. Thus although there can be no real laminar sub-layer once boiling has commenced, we can feel confident that there is always water next to some portions of the surface and this water is being heated by molecular conduction. We can think and speak of an effective laminar sub-layer. As the heat supplied to a surface is increased, the amount of bubbles increases but the surface temperature remains constant. There is a limit, however. When the bubbling becomes sufficiently vigorous, the heat transfer rate suddenly goes down and the temperature of the surface goes up until the surface melts. Another way of looking at equation (23) would be to say that only a very, very thin laminar sub-layer is necessary to keep the surface temperature no more than 15° above the saturation temperature at 1000 psia. Now we suppose the bubbling to get so vigorous as to reduce the thickness of even this layer. The value of the critical unstability is still the same, however, and to produce it a steeper gradient will be required. The wall temperature will rise and the bubbling continue and become more vigorous. Once started this process almost instantly proceeds to burn out.

Perhaps the most profitable use that this equation (22) can be put to is in connection with transients. If the temperature of a surface rises very rapidly with time, the temperature distribution in the water is obtained from the solution of the diffusion equation, considering that no natural convection can take place in such a short time and considering the water as a semi infinite medium. If the temperature rises lin-

early with time, the solution is of the form:

$$24.) \quad T = kt\left[\left(1 + \frac{z^2}{2\kappa t}\right) \operatorname{erfc} \frac{z}{2\sqrt{\kappa t}} - \frac{z}{\sqrt{\pi \kappa t}}\, e^{-\frac{z^2}{4\kappa t}}\right]$$

At some time both this equation and equation (22) will give the same value of $x_s$. $x_s$ is the value of z in equation (24) for which $T = T_s$. At this time the water next to the heated surface will begin to boil.

## 6. Summary.

Although this paper has some numbers in it, only the qualitative ideas are of any importance. For a particular combination of surface and liquid some critical amount of unstability is required in the liquid before vapor can form. If as in the case of highly turbulent flow of liquid over a heated surface, a temperature gradient exists through the liquid, one particular mass of the liquid will be the most likely to attain this degree of instability. Therefore, the bubbles of vapor formed will be of one size. If the amount of unstability required is known, or if the particular bubble size is known, a condition for boiling can be written. This simply requires that the distance that the saturation temperature reach into the liquid be inversely proportional to the cube root of the degrees of superheat of the surface. However, neither the distance, $x_s$, nor the superheat are independent variables. The equation can only be thought of as a limiting condition. If the laminar sub-layer is thick, the degrees of superheat required for boiling is determined by the critical unstability and the degrees

of superheat will remain constant until the layer is so reduced in thickness by bubble action that the surface temperature must rise to produce the critical unstability. If a surface is rising in temperature rapidly, then the equation can be interpreted to mean that boiling will not occur until the actual gradient in the water is the one required for the temperature of the surface at the moment.

## 7. Nomenclature

$E$ - internal energy-BTU

$Q$ - Heat quantity-BTU

$W$ - Work quantity-BTU

$H$ - Enthalpy-BTU

$M_g$ - Mass of saturated vapor-LBm

$s_g$ - Entropy per unit mass saturated vapor BTU/LBm-°R

$M_f$ - Mass of saturated liquid-LBm

$s_f$ - entropy per unit mass saturated liquid BTU/LBm-°R

$M$ - Total Mass of system LBm

$c_p$ - Constant pressure specific heat BTU/LBn-°F

$T$ - Temperature-°R

$T_w$ - Temperature of boundary (heated surface) °R

$T_s$ - Temperature of saturated liquid °R

$X_s$ - perpendicular distance from heated surface at which water temperature is $T_s$ -mils

$z$ - perpendicular distance from heated surface -mils

$R$ - Radius of hemisphere of liquid-mils

$s_{fg}$ - Sg-Sf

$h_{fg}$ - hg-h

$h_f$ - enthalpy per unit mass of saturated liquid BTU/LBn

$h_g$ - enthalpy per unit mass of saturated vapor BTU/LBm

$r$ - Radius of bubble - mils

$\rho_s$ - density of saturated liquid LBm/ft$^3$

$\rho_g$ - density of saturated vapor LBm/ft$^3$

$p''$ - Pressure of vapor

$p'$ — pressure of liquid

$C_1, C_2$ — radii of curvature

$\sigma$ — surface tension

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# APPENDIX

| Case | Pressure psia | Velocity ft/sec | Q/A BTU/ft$^2$-hr | Subcooling $^\circ$F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| A | 500 | 20 | $2.34 \times 10^6$ | 117 | 32.25 | 1.225 |
| | | | | | 493.25 | 2.453 |
| | | | | | 360.25 | 4.907 |
| | | | | | 170.5 | 7.351 |
| | | | | | 69.5 | 9.802 |
| | | | | | 23.25 | 12.251 |
| | | | | | 8.25 | 14.457 |
| | | | | | 1.5 | 17.152 |
| | | | | | 1.75 | 19.630 |
| | | | | | .2 | 22.053 |
| | | | | | .75 | 24.699 |
| B | 500 | 20 | $2.02 \times 10^6$ | 117 | 22.25 | 1.223 |
| | | | | | 445 | 2.447 |
| | | | | | 185.25 | 4.894 |
| | | | | | 150 | 7.345 |
| | | | | | 55.25 | 9.787 |
| | | | | | 10.5 | 12.233 |
| | | | | | 6.75 | 14.677 |
| | | | | | 3.5 | 17.127 |
| | | | | | 2.75 | 22.021 |
| | | | | | .5 | 29.361 |

| Case | Pressure psia | Velocity ft/sec | Q/A $BTU/ft^2$-hr | Subcooling °F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| C | 500 | 20 | $2.04 \times 10^6$ | 92 | 87.0 | 1.23 |
| | | | | | 743.2 | 2.46 |
| | | | | | 591.5 | 4.91 |
| | | | | | 121.0 | 7.36 |
| | | | | | 20.2 | 9.80 |
| | | | | | 3.2 | 12.30 |
| | | | | | .7 | 14.70 |
| D | 500 | 20 | $1.705 \times 10^6$ | 92 | 16.7 | 1.23 |
| | | | | | 512.2 | 2.46 |
| | | | | | 209.0 | 4.91 |
| | | | | | 40.5 | 7.36 |
| | | | | | 11.5 | 9.80 |
| | | | | | 5.0 | 12.30 |
| | | | | | 6.0 | 14.70 |
| | | | | | .5 | 17.20 |
| | | | | | .7 | 19.60 |
| | | | | | .2 | 24.60 |
| E | 500 | 20 | $1.43 \times 10^6$ | 92 | 12.5 | 1.23 |
| | | | | | 427.0 | 2.46 |
| | | | | | 186.5 | 4.91 |
| | | | | | 30.2 | 7.36 |
| | | | | | 10.2 | 9.80 |
| | | | | | 2.5 | 12.30 |
| | | | | | 3.5 | 14.70 |

| Case | Pressure psia | Velocity ft/sec | Q/A BTU/ft$^2$-hr | Subcooling $^\circ$F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| | | | | | 0.7 | 17.20 |
| | | | | | 1.2 | 19.60 |
| | | | | | .5 | 24.60 |
| F | 1000 | 20 | $2.155 \times 10^6$ | 95 | 44.7 | 1.26 |
| | | | | | 1277.3 | 2.50 |
| | | | | | 974.2 | 5.00 |
| | | | | | 232.3 | 7.600 |
| | | | | | 24.0 | 10.10 |
| | | | | | 5.0 | 12.60 |
| | | | | | .75 | 15.10 |
| | | | | | .75 | 17.60 |
| | | | | | .75 | 20.20 |
| G | 1000 | 20 | $1.820 \times 10^6$ | 95 | 336.8 | 1.26 |
| | | | | | 466.2 | 2.50 |
| | | | | | 179.1 | 5.00 |
| | | | | | 103.2 | 7.60 |
| | | | | | 98.8 | 10.10 |
| | | | | | 23.9 | 12.60 |
| | | | | | 9.0 | 15.10 |
| H | 1000 | 20 | $1.516 \times 10^6$ | 95 | 94.1 | 1.25 |
| | | | | | 467.8 | 2.50 |
| | | | | | 136.7 | 5.00 |
| | | | | | 3,.1 | 7.50 |

| Case | Pressure psia | Velocity ft/sec | Q/A BTU/ft$^2$-hr | Subcooling °F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| | | | | | 17.0 | 10.00 |
| | | | | | 0.56 | 12.50 |
| | | | | | 0.78 | 15.00 |
| I | 1000 | 20 | $1.258 \times 10^6$ | 70 | 344.9 | 1.26 |
| | | | | | 385.5 | 2.51 |
| | | | | | 348.1 | 5.02 |
| | | | | | 132.5 | 7.54 |
| | | | | | 98.3 | 10.05 |
| | | | | | 21.7 | 12.56 |
| | | | | | 6.5 | 15.07 |
| J | 1000 | 20 | $1.258 \times 10^6$ | 70 | 108.4 | 1.26 |
| | | | | | 546.0 | 2.50 |
| | | | | | 326.2 | 5.00 |
| | | | | | 159.5 | 7.60 |
| | | | | | 51.5 | 10.10 |
| | | | | | 7.5 | 12.60 |
| | | | | | 2.0 | 15.10 |
| | | | | | .9 | 17.60 |
| K | 1000 | 20 | $1.28 \times 10^6$ | 45 | 43.7 | 1.22 |
| | | | | | 488.5 | 2.45 |
| | | | | | 339.7 | 4.89 |
| | | | | | 198.2 | 7.34 |
| | | | | | 100.5 | 9.79 |

| Case | Pressure psia | Velocity ft/sec | Q/A BTU/ft$^2$-hr | Subcooling $^\circ$F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| | | | | | 46.7 | 12.24 |
| | | | | | 25.0 | 14.68 |
| | | | | | 15.75 | 17.13 |
| | | | | | 11.5 | 19.58 |
| | | | | | 9.0 | 22.02 |
| | | | | | 5.25 | 24.47 |
| | | | | | 1.25 | 26.92 |
| L | 1000 | 20 | $1.022 \times 10^6$ | 45 | 38.7 | 1.22 |
| | | | | | 496.5 | 2.45 |
| | | | | | 666.7 | 4.89 |
| | | | | | 72.5 | 7.34 |
| | | | | | 3.5 | 9.79 |
| M | 1000 | 20 | $.799 \times 10^6$ | 45 | 344.5 | 1.22 |
| | | | | | 530.7 | 2.45 |
| | | | | | 344.7 | 4.89 |
| | | | | | 96.0 | 7.34 |
| | | | | | 29.0 | 9.78 |
| | | | | | 1.5 | 12.23 |
| N | 1000 | 20 | $1.035 \times 10^6$ | 20 | 63 | 1.22 |
| | | | | | 364.0 | 2.43 |
| | | | | | 358.75 | 4.87 |
| | | | | | 292.5 | 7.30 |
| | | | | | 197.5 | 9.73 |

| Case | Pressure psia | Velocity ft/sec | $Q/A$ BTU/ft$^2$-hr | Subcooling $^\circ$F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| | | | | | 91.25 | 12.16 |
| | | | | | 37.50 | 14.60 |
| | | | | | 20.75 | 17.03 |
| | | | | | 13.25 | 19.46 |
| | | | | | 6.00 | 21.90 |
| | | | | | 4.00 | 24.33 |
| | | | | | 3.00 | 26.76 |
| | | | | | 1.5 | 29.20 |
| | | | | | .25 | 31.63 |
| | | | | | 0 | 34.06 |
| | | | | | 15 | 36.50 |
| O | 1000 | 20 | $.810 \times 10^6$ | 20 | 16.7 | 1.22 |
| | | | | | 308.7 | 2.45 |
| | | | | | 162.7 | 4.90 |
| | | | | | 169.0 | 7.35 |
| | | | | | 117.7 | 9.80 |
| | | | | | 93.5 | 12.25 |
| | | | | | 37.5 | 14.70 |
| | | | | | 23.0 | 17.15 |
| | | | | | 16.0 | 19.60 |
| | | | | | 11.0 | 22.05 |
| | | | | | 8.2 | 24.50 |
| P | 1000 | 20 | $.610 \times 10^6$ | 20 | 220.5 | 1.22 |
| | | | | | 372.75 | 2.44 |
| | | | | | 220.75 | 4.89 |

| Case | Pressure psia | Velocity ft/sec | Q/A BTU/ft$^2$-hr | Subcooling °F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| | | | | | 136.75 | 7.33 |
| | | | | | 80.75 | 9.77 |
| | | | | | 38.50 | 12.22 |
| | | | | | 18.75 | 14.66 |
| | | | | | 6.50 | 17.10 |
| | | | | | 2.75 | 19.54 |
| | | | | | .25 | 21.99 |
| Q | 1000 | 30 | $2.57 \times 10^6$ | 95 | 132.5 | 1.26 |
| | | | | | 751.0 | 2.51 |
| | | | | | 164.5 | 5.03 |
| | | | | | 47.5 | 7.55 |
| | | | | | 19.0 | 10.06 |
| | | | | | 3.0 | 12.58 |
| | | | | | 1.7 | 15.09 |
| R | 1000 | 30 | $2.22 \times 10^6$ | 97 | 433 | 1.26 |
| | | | | | 399.5 | 2.52 |
| | | | | | 180.2 | 5.04 |
| | | | | | 26.7 | 7.56 |
| | | | | | 7.7 | 10.03 |
| | | | | | 0.0 | 12.60 |
| | | | | | 0.2 | 15.12 |
| S | 1000 | 30 | $1.9 \times 10^6$ | 97 | 55.75 | 1.26 |
| | | | | | 203.5 | 2.52 |

| Case | Pressure psia | Velocity ft/sec | Q/A BTU/ft$^2$-hr | Subcooling °F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| | | | | | 90.75 | 5.04 |
| | | | | | 31.0 | 7,56 |
| | | | | | 10.75 | 10.08 |
| | | | | | 1.25 | 12.60 |
| | | | | | 2.00 | 15.12 |
| T | 1000 | 30 | $1.882 \times 10^6$ | 70 | 196.5 | 1.26 |
| | | | | | 985.3 | 2.52 |
| | | | | | 215.8 | 5.04 |
| | | | | | 31.7 | 7.56 |
| | | | | | 8.2 | 10.01 |
| | | | | | 1.5 | 12.60 |
| | | | | | 0.25 | 15.10 |
| U | 1000 | 30 | $1.545 \times 10^6$ | 70 | 14 | 1.29 |
| | | | | | 445 | 2.58 |
| | | | | | 111.5 | 5.17 |
| | | | | | 25.7 | 7.75 |
| | | | | | 1.7 | 10.30 |
| V | 1000 | 30 | $.846 \times 10^6$ | 20 | 119.2 | 1.22 |
| | | | | | 639.0 | 2.45 |
| | | | | | 232.0 | 4.90 |
| | | | | | 79.7 | 7.34 |
| | | | | | 16.2 | 9.79 |
| | | | | | 3.0 | 12.24 |

| Case | Pressure psia | Velocity ft/sec | q/A BTU/ft$^2$-hr | Subcooling °F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| | | | | | 1.5 | 14.69 |
| | | | | | .5 | 17.14 |
| W | 1000 | 30 | .614 x $10^6$ | 20 | 182 | 1.22 |
| | | | | | 355.7 | 2.45 |
| | | | | | 207.0 | 4.90 |
| | | | | | 69.5 | 7.35 |
| | | | | | 24.7 | 9.80 |
| X | 1000 | 30 | .452 x $10^6$ | 10 | 330.25 | 1.26 |
| | | | | | 362.00 | 2.52 |
| | | | | | 334.24 | 5.04 |
| | | | | | 182.75 | 7.57 |
| | | | | | 116.50 | 10.09 |
| | | | | | 22.50 | 12.62 |
| | | | | | 6.25 | 15.14 |
| Y | 1000 | 30 | .321 x $10^6$ | 7 | 291.5 | 1.25 |
| | | | | | 314.5 | 2.50 |
| | | | | | 260.0 | 5.00 |
| | | | | | 163.75 | 7.49 |
| | | | | | 104.0 | 9.99 |
| | | | | | 29.0 | 12.49 |
| | | | | | 5.75 | 14.99 |

| Case | Pressure psia | Velocity ft/sec | Q/A BTU/ft$^2$-hr | Subcooling °F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| Z | 1500 | 20 | $2.7 \times 10^6$ | 144 | 412 | 1.27 |
| | | | | | 641.2 | 2.54 |
| | | | | | 371.0 | 5.07 |
| | | | | | 72.7 | 7.60 |
| | | | | | 29.0 | 10.14 |
| | | | | | 0.7 | 12.68 |
| | | | | | 0.5 | 15.21 |
| AA | 1500 | 20 | $2.27 \times 10^6$ | 146 | 294.5 | 1.26 |
| | | | | | 617.75 | 2.53 |
| | | | | | 137.5 | 5.64 |
| | | | | | 25 | 7.58 |
| | | | | | 6.25 | 10.11 |
| | | | | | .25 | 12.63 |
| BB | 1500 | 20 | $1.97 \times 10^6$ | 94 | 10.7 | 1.26 |
| | | | | | 755.7 | 2.51 |
| | | | | | 222.5 | 5.02 |
| | | | | | 30.0 | 7.53 |
| | | | | | 5.2 | 10.04 |
| CC | 1000 | 20 | $1.669 \times 10^6$ | 69 | 21.7 | 1.26 |
| | | | | | 684.2 | 2.51 |
| | | | | | 249.2 | 5.02 |
| | | | | | 58.5 | 7.54 |

| Case | Pressure psia | Velocity ft/sec | Q/A BTU/$ft^2$-hr | Subcooling °F | Number | Diameter mils |
|---|---|---|---|---|---|---|
| | | | | | 21.2 | 10.00 |
| | | | | | 2.7 | 12.57 |
| | | | | | 1.5 | 15.10 |

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